U. G. Core Course - DSE 3 (SET 1) Semester - VI

Mathematics

LINEAR PROGRAMMING F.M - 75

Group - A (Compulsory)

Q (1.) $1 \times 10 = 10$

(a.) Define a convex set.

(b.) Define a convex hull.

(c.) Define slack and surplus variables in a L.P.P.

(d.) State the condition for standard form of the Primal L.P.P.

(e.) Define an assignment problem.

(f.) Define a transportation problem.

(g.) State reduction theorem in assignment problem.

(h.) What are the ways in which degeneracy occurs in a transportation problem.

(I.) Define basic feasible solution of a L.P.P.

(J.) Define dual of a L.P.P.

Q (2.)

(a.) Prove that a hyperplane is a convex set. $2\frac{1}{2}$

(b.) Prove that intersection of two convex sets is again a convex set. $2\frac{1}{2}$

Group - B

Answer any four :- $4 \times 15 = 60$

Q (3.) (a.) Using simplex method, solve the L.P.P.

$$\text{max } Z = 5x_1 + 3x_2$$

subject to

$$3x_1 + 5x_2 \leq 15$$

$$5x_1 + 2x_2 \leq 10$$

$$x_1 \geq 0 \; ; \; x_2 \geq 0$$

8

(b.) Prove that the dual of the dual of a given primal is the primal itself. 7

(4.) (a.) Write the dual of following L.P.P.

$$\text{Min. } Z = 3x_1 + x_2$$

Subject to $\quad 2x_1 + 3x_2 \geq 2$

$$x_1 + x_2 \geq 1 \; ; \; x_1 \geq 0 \; ; \; x_2 \geq 0$$

(7)

(b.) Prove that sphere is a convex set. (8)

(5.) (a.) Prove reduction theorem in connection with an assignment problem. (8)

(b.) Solve by Vogel's approximation method :- (7)

↓Factory↓	→Warehouse→ W_1	W_2	W_3	Supply
F_1	3	8	5	5
F_2	4	4	2	8
F_3	6	5	8	7
F_4	2	7	3	14
Demand	7	9	18	34

(6.) (a.) Find an optimal solution of the following assignment problem :- (8)

Jobs↓

Person →	A	B	C	D
1	87	85	71	38
2	91	89	75	34
3	70	72	86	75
4	37	35	21	88

(b.) Solve the following L.P.P. graphically :-

$$\text{Max } Z = 3x + 4y$$

Subject to $\quad x + y \leq 40$

$$x + 2y \leq 60$$

(7)

$$x \geq 0 \; ; \; y \geq 0$$

(7.)

(a.) Solve by two Phase method:-

Minimize $z = 3x_1 + 5x_2$

Subject to

$$6x_1 + 7x_2 \geq 11$$
$$4x_1 + 2x_2 \geq 13$$
$$x_1, x_2 \geq 0$$

(8)

(b.) write the dual of the following primal:

Maximize $z = 3x_1 + 2x_2 + x_3$

Subject to

$$2x_1 + x_2 - 3x_3 = 7$$
$$2x_1 + 3x_2 + 4x_3 = 9$$
$$x_1, x_2, x_3 \geq 0$$

(7)

(8.) (a.) Solve:-

Minimize $z = x_1 + 2x_2$

Subject to

$$2x_1 + 5x_2 \geq 6$$
$$x_1 + x_2 \geq 2$$
$$x_1, x_2 \geq 0$$

(7)

(b.) use duality to solve

minimize $z = 3y_1 + 2y_2$

Subject to

$$2y_1 + 3y_2 \geq 2$$
$$2y_1 + y_2 \geq 1$$
$$y_1, y_2 \geq 0$$

(8)

U.G. Core Course – DSE 4 (SET 1)

Semester – VI

Mathematics

F.M. 75

FLUID MECHANICS – I

Group – A (Compulsory)

$1 \times 10 = 10$

Q(1.) (a.) Define fluid pressure.

(b.) Define surfaces of equi-pressure.

(c.) Define centre of pressure of an area immersed in a fluid at rest under gravity.

(d.) What do you mean by resultant thrust.

(e.) State Archimedes Principle.

(f.) Define streamline and path line in a fluid motion.

(g.) Write down the equation of continuity in spherical polar co-ordinates.

(h.) Explain Eulerian method of fluid flow.

(I.) Explain Lagrangian method of fluid description.

(J.) Define Equation of continuity.

Q 2 Establish the formula $\frac{dp}{dz} = \rho \cdot g$ (5)

Group – B

Answer any four:- $4 \times 15 = 60$

Q 3 (a.) Liquids of densities ρ and ρ' occupy areas subtending angles α, α' at the centre of a fine vertical circular tube. If the surface of separation be at the lowest point then prove that

$$\sin\left(\frac{\alpha'}{2}\right) = \sqrt{\frac{\rho}{\rho'}} \cdot \sin\left(\frac{\alpha}{2}\right)$$ (8)

(b.) Three fluids whose densities are in A.P. fill a semi-circular tube whose bounding diameter is horizontal. Prove that the depth of one of the common surface is double of the others. (7)

(4.) (a.) A regular hexagon is immersed in water with one side in the surface. Find C.P. of the upper half. (8)

(b.) Find the C.P. of a circular area immersed vertically in liquid at rest under gravity with its centre at a depth 'h' from free surface. (7)

(5.) (a.) A circular cylinder of radius 'a' floats in a liquid at rest with its axis vertical and a length 'h' unimmersed. Show that if the cylinder is sufficiently long, it will float in equilibrium with its upper rim in the surface provided that the liquid is made to rotate with angular velocity $\frac{2\sqrt{gh}}{a}$. (8)

(b.) Discuss oscillation of water in a bent uniform tube in a vertical plane. (7)

(6.) (a.) Derive the equation of continuity in spherical polar co-ordinates. (8)

(b.) Show that in a two-dimensional incompressible steady flow fluid, the equation of continuity is satisfied with the velocity components in rectangular co-ordinates given by

$$u(x,y) = \frac{k\cdot(x^2-y^2)}{(x^2+y^2)^2} \; ; \; v(x,y) = \frac{2k\,x\cdot y}{(x^2+y^2)^2}$$ (7)

where 'K' is an arbitrary constant.

(7.) (a.) Establish the equation of continuity (Lagrangian form) (8)

(b.) Establish the equation of continuity in Cartesian co-ordinates. (7)

(8.) (a.) State and prove Bernoulli's theorem. (8)

(b.) Derive Euler's equation of motion. (7)

U.G. Core Course – C 13 (Set 1)

MECHANICS – II

Semester – VI

Mathematics

F.M. 75

Group – A (Compulsory)

$1 \times 10 = 10$

Q(1.) (a.) Define a common catenary.

(b.) State principle of virtual work.

(c.) State the energy test of stability.

(d.) Define stable and unstable equilibrium.

(e.) Define pitch and screw.

(f.) Define central orbit.

(g.) What is an apse and apsidal angle.

(h.) State Kepler's laws.

(I.) State D' Alembert's Principle

(J.) Define simple equivalent pendulum.

Q(2.) Show that in a catenary

(i) $s = c\tan\psi$ (ii) $s = c\sinh\left(\frac{x}{c}\right)$ (5)

Group – B

Answer any four :-

$4 \times 15 = 60$

Q(3.) (a.) Find the equations of the central axis of any given system of forces. (8)

(b.) State and prove principle of virtual work. (7)

Q(4.) (a.) A uniform chain of length 'l' is to be suspended from two points A and B in the same horizontal line so that either terminal tension is 'n' times that at the lowest point. Prove that the span AB must be

$$\frac{l}{\sqrt{n^2-1}} \cdot \log_e\left(n+\sqrt{n^2+1}\right)$$ (8)

(4.) (b.) A hemisphere rests in equilibrium on a sphere of equal radius. Show that the equilibrium is unstable when the curved and stable when the flat surface of the hemisphere rests on the sphere. (7)

(5.) (a.) Find the Resultant Wrench of two given wrenches. (8)

(b.) Discuss the conditions of equilibrium of a rigid body which has two points A and B fixed so that the body can turn about the fixed axis AB. (7)

(6.) (a.) A Particle moves with a central acceleration $\frac{\mu}{(\text{distance})^3}$. Find the Path and discuss three cases that arise. (8)

(b.) obtain differential equation of central orbit in (i) Polar form (ii) Pedal form (7)

(7.) (a.) Prove that the motion of a body about its centre of inertia is the same as it would be if the centre of inertia were fixed and the same forces acted on the body. (8)

(b.) If h and h' be the greatest heights in the two paths of a projectile for a given range 'R' then show that $R = 4.h.h'$ (7)

(8.) (a.) Deduce Kepler's first law of planetary motion from Newton's law of gravitation. (8)

(b.) A uniform rod of mass 'm' and length $2a$ can turn freely about one end which is fixed; it is started with angular velocity ω from the position in which it hangs vertically, find the motion. (7)

U.G. Core Course – C 14 (SET 1) Semester – VI

Mathematics

NUMERICAL ANALYSIS

F.M – 75

Group – A (Compulsory)

Q(1.)

$1 \times 10 = 10$

(a.) Write Newton–Raphson's formula for finding a real root of $f(x) = 0$

(b.) State the condition when Newton–Raphson's method fails.

(c.) If Δ and ∇ are forward and backward difference operators, Prove that $\Delta - \nabla = \Delta \cdot \nabla$

(d.) Show that $\Delta \log f(x) = \log\left(1 + \frac{\Delta f(x)}{f(x)}\right)$

(e.) Write the Simpson's $\frac{1}{3}$rd rule for numerical integration.

(f.) Write the Simpson's $\frac{3}{8}$th rule for numerical integration.

(g.) Evaluate $\left(\frac{\Delta^2}{E}\right)(x^2)$

(h.) Prove that $(1+\Delta)\cdot(1-\nabla) = 1$ where Δ and ∇ are the first forward and backward difference operators of $f(x)$.

(I.) Express $2x^3 - 3x^2 + 3x - 10$ in factorial notation, the interval of differencing being unity.

(J.) Write Picard's method of successive approximation of differential equation $\frac{dy}{dx} = f(x,y)$; $y = y_0$ when $x = x_0$.

Q (2.)

(a.) Find a real root of the equation $x^3 - x + 1 = 0$ using Bisection method. (3)

(b.) Write Lagrange's interpolation formula for unequal intervals. (2)

Group – B

$4 \times 15 = 60$

Answer any four:–

Q (3.) (a.) Find cube root of 17 using Newton–Raphson's method. (7)

(3.) (b.) Find a real root of $x^3 - 9x + 1 = 0$ by Regula Falsi method. (8)

(4.) (a.) obtain the missing figure in the following:- (8)

x	1	2	3	4	5	6	7	8
y	1	8	—	64	—	216	343	512

(b.) Find $f(8)$ and $f(15)$ from following table by means of Newton's divided difference formula:- (7)

x	4	5	7	10	11	13
y	48	100	294	900	1200	2028

(5.) (a.) Derive Newton's divided difference formula for unequal intervals. ($7\frac{1}{2}$)

(b.) Derive Lagrange's interpolation formula for unequal intervals. ($7\frac{1}{2}$)

(6.) (a.) Discuss regula falsi method for finding a real root of $f(x) = 0$. ($7\frac{1}{2}$)

(b.) Discuss Newton-Raphson's method for finding a real root of $f(x) = 0$ ($7\frac{1}{2}$)

(7.) (a.) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rules. (8)

(b.) Derive Newton-Cote's Quadrature formula. (7)

(8.) (a.) Find $f'(5)$ from the following table:-

x	0	2	3	4	7	9
$f(x)$	4	26	58	112	466	922

(8)

(b.) Using Picard's method, solve $\frac{dy}{dx} = 1 - 2xy$; $y(0) = 0$ upto the second approximation. (7)