P.G. Course Paper No – ECMAT401A

M.Sc. Semester – 4

Mathematics

F.M.: – 70

Optimization Techniques

Group A (Compulsory)

Q(1) $1 \times 5 = 5$

(a.) What is the difference between simplex and dual simplex method.

(b.) Define saddle point.

(c.) State maximin principle.

(d.) State minimax principle.

(e.) Write basic characteristics of queueing system.

Q(2)

(a.) What is the advantage of dual simplex method over simplex method. $2\frac{1}{2}$

(b.) Write characteristics of game theory. $2\frac{1}{2}$

Group-B

Answer any four:- $4 \times 15 = 60$

Q(3) Solve using Dual Simplex method:-

$$\text{Minimize } Z = 3y_1 + 2y_2$$

$$\text{Subject to } \quad 2y_1 + 3y_2 \geq 2$$

$$2y_1 + y_2 \geq 1$$

$$y_1, y_2 \geq 0$$

Q(4) Discuss by an example, the changes occur in the structure of the LPP due to addition of new variable/ Deleting of existing variable/ Addition of New constraints/ Deletion of existing constraints.

(5.) Solve the game whose pay-off matrix is

Player Y

$$\text{Player } X \quad \begin{array}{c|ccc} & Y_1 & Y_2 & Y_3 \\ \hline X_1 & -3 & -2 & 6 \\ X_2 & 2 & 0 & 2 \\ X_3 & 5 & -2 & -4 \end{array}$$

(6.) Two companies A and B are competing for the same product. Their different strategies are given by the following pay-off matrix:

Company B

$$\text{Company } A \quad \begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{bmatrix} 2 & -2 & 3 \\ -3 & 5 & -1 \end{bmatrix} \end{array}$$

Find the optimum strategies for both the companies and also the value of the game by Linear Programming method.

(7.) Solve the following game by using simplex method:-

Player B

$$\text{Player } A \quad \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{bmatrix} 9 & 1 & 4 \\ 0 & 6 & 3 \\ 5 & 2 & 8 \end{bmatrix} \end{array}$$

(8.) Discuss steady state solution of Markovian queueing model M/M/1, M/M/1 with limited waiting space.

Additional questions M.Sc. Sem - 4

Problems on Sensitivity Analysis. Solve them. Mathematics

Q① A Company produce three Products: A, B and C. Each Product requires two raw materials: Steel and aluminium. The following Linear Programming model describes the Company's Product mix Problem.

$$\text{Max } Z = 30x_A + 10x_B + 50x_C$$

Subject to
$$6x_A + 3x_B + 5x_C \leq 450 \text{ (Steel)}$$
$$3x_A + 4x_B + 5x_C \leq 300 \text{ (Aluminium)}$$
and $x_A, x_B, x_C \geq 0$

The optical Product plan is the following table where S_S and S_A are the slack variables for unused steel and aluminium quantity respectively.

		$c_j \rightarrow$	30	10	50	0	0
c_B	B	$b = x_B$	x_A	x_B	x_C	S_S	S_A
0	S_S	150	3	-1	0	1	-1
50	x_C	60	3/5	4/5	1	0	1/5
Z=16		z_j	30	40	50	0	10
		$\Delta_j = c_j - z_j$	0	-30	0	0	-10

(a.) Determine the Sensitivity limits for the available steel and aluminium within which the present product mix will remain optimal.

(b.) Find the new optimal solution when available steel is 300 tonnes and aluminium is 400 tonnes.

Q② Given the optimal Solution in the following table, discuss the effect on optimality by adding a new variable with column coefficients (3, 3, 3) and coefficient 5 in the objective function (minimization)

		$C_j \rightarrow$	3	8	0	0	M
C_B	B	$b(=x_B)$	x_1	x_2	S_1	S_2	A_1
0	S_2	60	0	0	-1	1	1
3	x_1	80	1	0	1	0	0
8	x_2	120	0	1	-1	0	1
$Z=1200$		Z_j	3	8	-5	0	8
		$\Delta_j = C_j - Z_j$	0	0	5	0	M-8

Q③ Given the optimal solution in the following table, discuss (a.) effect of change in C_3 of non-basic variable x_3 (b.) effect of change in the coefficient of C_1 of basic variable x_1.

		$C_j \rightarrow$	4	6	2	0	0
C_B	B	$b(=x_B)$	x_1	x_2	x_3	S_1	S_2
4	x_1	1	1	0	-1	4/3	-1/3
6	x_2	2	0	1	2	-1/5	1/3
$Z=16$	B	Z_j	4	6	8	10/3	2/3
		$C_j - Z_j$	0	0	-6	$-\frac{10}{3}$	-2/3

Ans
Solution to Q③:- (a.) $-\infty < C_3 \leq 8$

(b.) $\frac{3}{2} \leq C_1 \leq 6$

Solution to Q②:- $x_1 = 20$, $x_2 = 120$, $x_7 = 20$; $\min. Z = 1120$