U.G. Core Course – C 4 (SET-I) SEMESTER – II
SUBSECT – Mathematics
F.M – (75)

Integral Calculus and Analytic Geometry 3D

Group – A (Compulsory)

$1 \times 10 = (10)$

Q (1.) (a.) Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

(b.) Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

(c.) write the formula for $\int \sqrt{x^2+a^2}\,dx$

(d.) Define Point of inflexion

(e.) Define double Point.

(f.) write the relation between rectangular and polar Cylindrical Co-ordinates.

(g.) Define direction Cosines.

(h.) Define a Plane.

(I.) write the relation between rectangular and polar Spherical Co-ordinates.

(J.) Define skew lines.

Q (2) (a.) Trace the curve $x^2+y^2=a^2$ (3)

(b.) write the equation of the sphere Passing through the Points $(0,0,0)$, $(a,0,0)$, $(0,b,0)$, $(0,0,c)$ (2)

Group – B

Answer any four :– $4 \times 15 = (60)$

Q (3) (a.) Evaluate $\int_0^\infty e^{-x^2} dx$ $(7\frac{1}{2})$

(b.) find the reduction formula for $\int \sin^m x \cdot \cos nx\,dx$ $(7\frac{1}{2})$

(4.) (a.) Trace the curve $x^3+y^3=3axy$ and find the length of its loop. (7½)

(b.) Find the area bounded by the curve $x^{2/3}+y^{2/3}=a^{2/3}$ (7½)

(5.) (a.) Find the area of the cardioid $r=a.(1+\cos\theta)$ (7½)

(b.) Find the volume and surface area of the solid obtained by revolving the circle $x^2+y^2=a^2$ about x-axis. (7½)

(6.) (a.) Derive the expression $\cos\theta = l_1l_2+m_1m_2+n_1n_2$ where θ is the angle between two straight lines whose direction cosines are respectively l_1, m_1, n_1 and l_2, m_2, n_2. (7½)

(b.) If lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ intersect, then find their point of intersection. (7½)

(7.) (a.) Find the shortest distance and the cartesian equation of line of shortest distance between following pair of lines:- $\frac{x-1}{2}=\frac{y+1}{3}=z$ and $\frac{x+1}{3}=\frac{y-2}{1}$; $z=2$ (7½)

(b.) Derive the equation of the plane in intercept form (7½)

(8.) (a.) Find the equation of the sphere which passes through four points $(0,0,0)$, $(0,1,-1)$, $(-1,2,0)$ and $(1,2,3)$ (7½)

(b.) Prove that the plane $x+2y-z=4$ cuts the sphere $x^2+y^2+z^2-x+z-2=0$ in a circle of radius unity. Also, find the co-ordinates of the centre of the circle. (7½)