U.G. Core Course - C3 (SET-I) (Semester - II) (Subject - Mathematics)

ANALYSIS-I

F.M - 75

Group-A (Compulsory)

Q(1.) $1 \times 10 = 10$

(a.) Write the completeness property of R, the set of real numbers.

(b.) Write Archimedean property in R, the set of real numbers.

(c.) Define neighbourhood of a point in a given set.

(d.) Define limit point of a set.

(e.) Define open sets and closed sets.

(f.) State Bolzano - Weierstrass theorem.

(g.) Define a Cauchy sequence

(h.) State Cauchy's root test.

(I.) Define absolutely convergent series and conditionally convergent series.

(J.) State Cauchy's integral test.

Q(2.) State and Prove Pringsheim's theorem. $1 \times 5 = 5$

Group-B

Answer any four questions:- $4 \times 15 = \boxed{60}$

Q(3.) (a.) Prove that a set 'A' is closed iff its complement is open. $7\frac{1}{2}$

(b.) Prove that intersection of finite number of open sets is always an open set but intersection of infinite number of open sets is not necessarily open. $7\frac{1}{2}$

Q(4.) (a.) Prove that a convergent sequence is always bounded but its converse is not necessarily true. $7\frac{1}{2}$

(b.) Prove that every monotonic increasing sequence tends to its least upper bound. $7\frac{1}{2}$

(5.) (a.) State and Prove Cauchy's general Principle of Convergence. $7\frac{1}{2}$

(b) Using Cauchy's general Principle of Convergence, Prove that the sequence $\{a_n\} = \{1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}\}$ is not Convergent. $7\frac{1}{2}$

(6.) (a.) Prove that the series $\sum \frac{1}{n^p}$ is convergent if $p>1$ and divergent if $p \leq 1$. $7\frac{1}{2}$

(b.) Test the Convergence of the series

$$\sum u_n = \sum \frac{n^2-1}{n^2+1} \cdot x^n \; ; x>0$$ $7\frac{1}{2}$

(7.) (a.) State and Prove Raabe's test. $7\frac{1}{2}$

(b.) Test the Convergence of the series

$$\sum u_n = \sum_{n=3}^{\infty} \frac{1}{n \cdot \log n \cdot (\log \log n)^p}$$ $7\frac{1}{2}$

(8.) (a.) Prove that every absolutely Convergent series is always convergent but its converse need not be always true. $7\frac{1}{2}$

(b.) Test the Convergence of the series

$$\sum a_n = \sum \frac{1^2 \cdot 3^2 \cdot \cdots \cdot (2n-1)^2}{2^2 \cdot 4^2 \cdot \cdots \cdot (2n)^2} \cdot x^{n-1} \; ; x>0$$ $7\frac{1}{2}$